A Possible Reinterpretation of Einstein's Equations

A. Bouda · A. Belabbas

Abstract In this paper, we first review Huei's formulation in which it is shown that the linearized Einstein equations can be written in the same form as the Maxwell equations. We eliminate some imperfections like the scalar potential which is ill linked to the electric-type field, the Lorentz-type force which is obtained with a time independence restriction and the undesired factor 4 which appears in the magnetic-type part. Second, from these results and in the light of a recent work by C.C. Barros, we propose an extension of the equivalence principle and we suggest a new interpretation for Einstein's equations by showing that the electromagnetic Maxwell equations can be derived from a new version of Einstein's ones.

Keywords Linearized Einstein's equations · Principle of equivalence · Maxwell's equations · Lorentz force

1 Introduction

There are several attempts to describe gravitation and electromagnetism in a unified field theory. The electromagnetic field was sometimes presented as the nonsymmetric part of the metric [1, 2], sometimes as some additional components of the metric in five dimensional space-time [3]. In this paper, we suggest that the electromagnetic field is contained in the four dimensional Einstein equations themselves. Our approach is based on two things.

The first is Huei's formulation [4] in which it is shown that the linearized Einstein equations can be written in the same form as the Maxwell equations. In this formulation, developed also by Wald [5], there are however some imperfections to point out:

1. The usual relation between the potential and the electric-type field as known in electromagnetism is reproduced only in the harmonic gauge.

A. Bouda (✉) · A. Belabbas
Laboratoire de Physique Théorique, Université de Béjaïa, Route Targa Ouazemour, 06000 Béjaïa, Algeria
e-mail: bouda_a@yahoo.fr

A. Belabbas
e-mail: belabbas.moumene@gmail.com

2. The geodesic equation predicts a Lorentz-type force with an undesired factor 4 in the magnetic-type part compared to the usual electromagnetic Lorentz force.
3. The Lorentz-type force is obtained only in the case where the fields are independent on time.

We would like to add that Carroll [6] has redefined the fields in such a way as to eliminate the undesired factor 4. However, Maxwell-type equations are not satisfied in his formulation. In what follows, with the use of a subtle gauge, we will review the linearized gravity so as to get to a strong similarity with electromagnetism.

The second thing is the recent work by C.C. Barros in which he suggested to study the effect of the metric in the subatomic systems instead of trying to quantize gravity [7–9]. He assumed that interactions, even non gravitational ones, affect the space-time structure. In the context of Schwarzschild's solution, by introducing Coulomb's potential instead of the gravitational one into the metric, he derived the spectrum of hydrogen atom. Although the author considered a subatomic system, however the use of Schwarzschild's solution implies that he implicitly assumed the existence of versions of Einstein's equations and geodesic equations for the Coulomb interaction. What is even more surprising is that his results for hydrogen atom spectrum, confirmed in more detail in [10], are extremely close to those predicted by Dirac's equation. These intriguing results encourage us to wonder about the extension of the principle of equivalence (PE).

The Barros results and the extraordinary similarity between gravitation and electromagnetism resulting from the revised version of linearized gravity presented here suggest that the electromagnetic Maxwell equations can be followed from a new version of Einstein's equations in the linear approximation within the framework of the electromagnetic interaction. We will see that the higher order terms of this new version of Einstein's equations are negligible in the usual application domains of the electromagnetism.

The paper is organized as follows. In Sect. 2, we will review the linearized gravity. In Sect. 3, we will present several arguments to show that the current version of the PE is restrictive and suggest how it can be extended to include other interactions. We consolidate our arguments by showing in the context of the electromagnetic interaction that Maxwell's equations are contained as a first order approximation in Einstein's equations. Section 4 is devoted to conclusion.

2 Linearized Gravity Revisited

Our goal here is to review the linearized gravity so as to avoid the imperfection cited above and then to get to a strong similarity with electromagnetism. For this purpose, let us decompose the metric into the Minkowski one plus a perturbation

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{1}$$

where $\eta_{\mu\nu} = (+1, -1, -1, -1)$ and $h_{\mu\nu} \ll 1$. To first order, setting

$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} \tag{2}$$

and taking into account (1), it is easy to check that indices of $h^{\mu\nu}$ can be raised and lowered by using $\eta_{\mu\nu}$ and $\eta^{\mu\nu}$. In this approximation, the Christoffel symbols,

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\rho} \left[\partial_{\mu} g_{\rho\nu} + \partial_{\nu} g_{\mu\rho} - \partial_{\rho} g_{\mu\nu} \right], \tag{3}$$

and the curvature tensor,

$$R^{\lambda}{}_{\mu\nu\rho} = \partial_{\rho}\Gamma^{\lambda}_{\mu\nu} - \partial_{\nu}\Gamma^{\lambda}_{\mu\rho} + \Gamma^{\sigma}_{\mu\nu}\Gamma^{\lambda}_{\rho\sigma} - \Gamma^{\sigma}_{\mu\rho}\Gamma^{\lambda}_{\nu\sigma}, \tag{4}$$

take the forms

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}\eta^{\lambda\rho}\left[\partial_{\mu}h_{\rho\nu} + \partial_{\nu}h_{\mu\rho} - \partial_{\rho}h_{\mu\nu}\right] \tag{5}$$

and

$$R_{\lambda\mu\nu\rho} = \frac{1}{2}\left[\partial_{\rho}\partial_{\mu}h_{\lambda\nu} + \partial_{\nu}\partial_{\lambda}h_{\mu\rho} - \partial_{\lambda}\partial_{\rho}h_{\mu\nu} - \partial_{\mu}\partial_{\nu}h_{\lambda\rho}\right]. \tag{6}$$

The infinitesimal coordinate transformation $x^{\mu} \rightarrow x'^{\mu} = x^{\mu} + \xi^{\mu}$ induces on the perturbation of the metric a transformation $\delta h_{\mu\nu} = -\partial_{\nu}\xi^{\lambda}\eta_{\mu\lambda} - \partial_{\mu}\xi^{\lambda}\eta_{\nu\lambda}$ which leaves invariant expression (6) of the curvature tensor. This invariance allows us to fix a gauge

$$g^{\mu\nu}\Gamma^{\rho}_{\mu\nu} = 0, \tag{7}$$

known as the harmonic gauge [5, 6, 11]. In the linear approximation, this condition takes the following form

$$\partial_{\mu}h^{\mu}_{\nu} - \frac{1}{2}\partial_{\nu}h = 0, \tag{8}$$

where $h \equiv h^{\mu}_{\mu}$ is the trace of the perturbation. Huei defined the electric-type field and the potential vector as

$$E^{i}_{g} = \frac{c^2}{4}\left[\partial^{i}\hat{h}^{00} + \partial_{j}\hat{h}^{ij}\right] \tag{9}$$

and

$$A^{i}_{g} = \frac{c}{4}\hat{h}^{0i}, \tag{10}$$

where $\hat{h}^{\mu\nu} = h^{\mu\nu} - h\eta^{\mu\nu}/2$, c is the velocity of light and $i, j, \ldots = 1, 2, 3$. It is assumed that the Einstein convention for repeated indexes works also for i and j. In this formulation, although expression (9) satisfies Maxwell-type equations, however it does not allow to reproduce the expected relation

$$\vec{E}_{g} = -\vec{\nabla}\phi - \frac{\partial\vec{A}_{g}}{\partial t}, \tag{11}$$

where ϕ is the Newtonian potential. Nevertheless, if we use the gauge condition (8), expression (9) turns out to be

$$E^{i}_{g} = \frac{c^2}{4}\partial^{i}\hat{h}^{00} - \frac{c^2}{4}\partial_{0}\hat{h}^{0i}, \tag{12}$$

which is compatible with (10) and (11) if we set $\phi = c^2\hat{h}_{00}/4$. This feature is unsatisfactory since relation (11) should be valid in any gauge. We also notice that in the Lorentz-type force,

$$\frac{d^2\vec{r}}{dt^2} = \vec{E}_{g} + 4\vec{v} \times \vec{B}_{g}, \tag{13}$$

obtained from the geodesic equation, there is in the magnetic-type part an undesired factor 4 compared to the electromagnetic case. In addition, this equation is obtained only if the fields do not depend on time.

In what follows, we will remedy these weakness by defining the scalar and vector potentials as

$$A_g^0 = \frac{c}{2} h^{00},$$ (14)

$$A_g^i = c h^{0i}$$ (15)

and introducing the tensor

$$F_g^{\mu\nu} = \partial^\mu A_g^\nu - \partial^\nu A_g^\mu,$$ (16)

as in electromagnetism. We notice that definition (14) of the scalar potential is compatible with the well-known expression obtained when the geodesic equation is used in the Newtonian approximation. Contrary to Huei's formulation, by defining $E_g^i = -c F_g^{0i}$ as in electromagnetism, relation (11) is automatically satisfied without using any gauge. It is also the case for

$$\vec{B}_g = \vec{\nabla} \times \vec{A}_g,$$ (17)

where $B_g^i = -\epsilon^{ijk} (F_g)_{jk}/2$ is the magnetic-type field and ϵ^{ijk} is the usual Levi-Civita antisymmetric tensor ($\epsilon^{123} = +1$). Concerning the first group of the Maxwell-type equations,

$$\partial^\lambda F_g^{\mu\nu} + \partial^\nu F_g^{\lambda\mu} + \partial^\mu F_g^{\nu\lambda} = 0,$$ (18)

with the use of definition (16), it is automatically satisfied. With regard to the second group, let us consider the Einstein tensor

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$ (19)

and use the following definition for the Ricci tensor

$$R_{\mu\nu} = g^{\lambda\rho} R_{\lambda\mu\rho\nu}.$$ (20)

In the linear approximation, by using (1), (2), (6) and (20), expression (19) turns out to be

$$G_{\mu\nu} = \frac{1}{2} \big[\partial_\mu \partial_\nu h + \Box h_{\mu\nu} - \eta_{\mu\nu} \Box h. - \partial_\lambda \partial_\mu h_\nu^\lambda - \partial_\lambda \partial_\nu h_\mu^\lambda + \eta_{\mu\nu} \partial_\rho \partial_\lambda h^{\rho\lambda} \big],$$ (21)

where $\Box = \partial^\lambda \partial_\lambda$ is the d'Alembertian. Carroll [6] showed that the trace $h^i_{\ i}$ is not a propagating degree of freedom. Then, instead of the time component of (8), let us impose an alternative condition

$$h^i_{\ i} = 0$$ (22)

and keep in the gauge condition (8) only the three space components

$$\partial_\mu h_i^\mu - \frac{1}{2} \partial_i h = 0.$$ (23)

From relation (22), we observe that $h = h_{00}$ and expressions (14) and (15) can be written in a unified form

$$A_g^\mu = c \left(h^{0\mu} - \frac{1}{2} \eta^{0\mu} h \right). \tag{24}$$

With the use of (22), we can deduce from (21) that

$$G_{00} = \frac{1}{2} \partial_i \partial_j h^{ij} \tag{25}$$

and

$$G_{0i} = \frac{1}{2} \left[\partial_j \partial^j h_{0i} - \partial_0 \partial^j h_{ij} - \partial_i \partial^j h_{0j} \right]. \tag{26}$$

Relations (22) and (23) allow to write (25) and (26) as follows

$$G^{00} = \frac{1}{2} \partial_i \left[\frac{1}{2} \partial^i h^{00} - \partial^0 h^{0i} \right] \tag{27}$$

and

$$G^{0i} = \frac{1}{2} \left[\partial_j \left(\partial^j h^{0i} - \partial^i h^{0j} \right) + \partial_0 \left(\partial^0 h^{0i} - \frac{1}{2} \partial^i h^{00} \right) \right]. \tag{28}$$

Taking into account (16) and (24), relations (27) and (28) become

$$G^{00} = \frac{1}{2c} \partial_i F_g^{i0} = \frac{1}{2c} \partial_\mu F_g^{\mu 0} \tag{29}$$

and

$$G^{0i} = \frac{1}{2c} \partial_\mu F_g^{\mu i}. \tag{30}$$

Finally, we have

$$G^{0\nu} = \frac{1}{2c} \partial_\mu F_g^{\mu \nu}. \tag{31}$$

It is clear that in the vacuum, Einstein's equations, $G^{0\nu} = 0$, reduce to the Maxwell-type equations $\partial_\mu F_g^{\mu \nu} = 0$. We notice that the time component of (8), which has not been used, represents the Lorentz-type gauge $\partial_\mu A_g^\mu = 0$. The last point concerns the geodesic equation

$$\frac{d^2 x^\lambda}{d\tau^2} + \Gamma_{\mu\nu}^\lambda \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0, \tag{32}$$

which allows us to write in the linear approximation

$$\frac{d^2 x^i}{d\tau^2} = c^2 \left[\frac{1}{2} \left(\frac{dt}{d\tau} \right)^2 \partial^i h^{00} - \frac{1}{c} \left(\frac{dt}{d\tau} \right)^2 \frac{\partial h^{0i}}{\partial t} \right.$$
$$\left. - \frac{u_j}{c} \frac{dt}{d\tau} \left(\partial^j h^{0i} - \partial^i h^{0j} \right) - \frac{u_j}{c^2} \frac{dt}{d\tau} \frac{\partial h^{ij}}{\partial t} - \frac{u_j u_k}{c^2} \left(\partial^j h^{ik} - \frac{1}{2} \partial^i h^{jk} \right) \right], \tag{33}$$

τ being the proper time and u_ν a component of the four-velocity. For lower velocities, we have $dt/d\tau \rightarrow 1$ and $u^i \rightarrow v^i = dx^i/dt$. Then, if we neglect the terms proportional to $1/c^2$ in (33) and use (24), we get to

$$\frac{d^2 x^i}{dt^2} = \left(c\, \partial^i A_g^0 - \frac{\partial A_g^i}{\partial t} \right) - v_j \left(\partial^j A_g^i - \partial^i A_g^j \right), \tag{34}$$

from which we deduce that

$$\frac{d^2 \vec{r}}{dt^2} = \vec{E}_g + \vec{v} \times \vec{B}_g. \tag{35}$$

We emphasize that this result is obtained without imposing the time independence restriction for the fields as the case in [4, 5]. Furthermore, the undesired factor 4 in the magnetic-type part disappears.

3 Maxwell's Equations from the Einstein Ones

This extraordinary similarity between gravitation and electromagnetism obtained in the last section prompt us to wonder about the extension of the PE. In fact, the PE states that at every space-time point in an arbitrary gravitational field it is possible to choose a "locally inertial coordinate system" such that, within a sufficiently small region of the point in question, the laws of nature take the same form as in unaccelerated Cartesian coordinate systems [11]. Let us provide further arguments in favor of an extension of the PE.

First, we think that there is some vagueness about the expression "sufficiently small region". In fact, Cartesian coordinate system implies vanishing values for the Christoffel symbols $\Gamma_{\mu\nu}^\lambda$. From a geometrical point of view, it is well-known that at any point it is possible to impose a vanishing value for the $\Gamma_{\mu\nu}^\lambda$ by a suitable coordinate system. The non-vanishing value of the curvature tensor, expression (4), is then ensured by the derivatives of $\Gamma_{\mu\nu}^\lambda$. However, if we substitute the point by a small region, the vanishing values of $\Gamma_{\mu\nu}^\lambda$ will impose also vanishing values for their derivatives in this region and then for the curvature tensor, which must keep this value in any coordinate system. This conclusion is absurd since we assumed from the start the presence of the gravitational field. Finally, to be precise, the PE works just at a point and not at a "small region" providing then the possibility of canceling any interaction, even the non gravitational one, by a suitable coordinate system.

Second, even if we consider the equality between the inertial mass and the gravitational one is perfectly exact, two bodies A and B do not fall to Earth with the same acceleration in the same frame. In fact, the same acceleration can be measured for the two bodies in two different frames associated with the mass centers of each body and the Earth. It is only in the case of the same mass for A and B or in the case where each body mass is negligible with respect to the Earth one that the two mass centers coincide. This means that the property from which all bodies fall with the same acceleration is just an approximation which works in particular circumstances.

The third argument is the Einstein unified theory [1, 2]. Although he presented the PE as the cornerstone upon which general relativity is based, he attempted to extend his theory in order to include the electromagnetic field without concerning himself with the PE. That's one way of admitting that the PE can be extended to other interactions.

The last and main argument is the Barros [7–9] result concerning the hydrogen atom. He reproduced its spectrum by introducing Coulomb's potential into the metric. Because of the

spherical symmetry, he used the Schwarzschild solution

$$ds^2 = \xi_e c^2 dt^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) - \xi_e^{-1} dr^2, \tag{36}$$

where $\xi_e(r)$ is determined by Coulomb's interaction $V(r) = -Ke^2/r$, and he showed that for weak potential

$$\xi_e(r) = 1 - \frac{2K}{c^2}\frac{e}{r}\frac{e}{m_e}, \tag{37}$$

m_e being the electron mass. Observe that in the gravitational case we have

$$\xi_g(r) = 1 - \frac{2G}{c^2}\frac{M}{r}\frac{m_g}{m_i}, \tag{38}$$

in which the ratio between the gravitational and the inertial masses is given by $m_g/m_i = 1$. The author tacitly assumed that the above expression of ds^2 follows from the Einstein equation version of electromagnetic field. This suggests that a particle motion under the electric force can be described by a geodesic equation. In fact, let us consider a charge q of mass m under the action of a fixed charge Q. In the weak field case and lower velocities $v \ll c$, we can deduce from (32) that

$$\frac{d^2\vec{r}}{dt^2} = -\frac{c^2}{2}\vec{\nabla}h_{00}. \tag{39}$$

If we set

$$h_{00} = \frac{2K}{c^2}\frac{Qq}{m}\frac{1}{r}, \tag{40}$$

we reproduce the motion equation in agreement with the second Newton law. The linear approximation is used here and will be justified below.

All these arguments suggest us two things. The first concerns the possible extension of the PE in such away as to remove the action of any fundamental interaction by a suitable coordinate system. We will come back to this question in the last section. The second suggestion is that Maxwell's equations are contained as a first order approximation in a new version of Einstein's ones corresponding to the electromagnetic context. In fact, as in gravity, let us define in presence of a test particle of mass m and charge q the four-vector potential as

$$A^\mu = \frac{mc}{q}\left(h^{0\mu} - \frac{1}{2}\eta^{0\mu}h\right), \tag{41}$$

where $h^{\mu\nu}$ satisfy conditions (22) and (23) as in above. The well-known definition of the electromagnetic tensor, $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, guarantees automatically the first group of Maxwell's equations,

$$\partial_\lambda F_{\mu\nu} + \partial_\nu F_{\lambda\mu} + \partial_\mu F_{\nu\lambda} = 0, \tag{42}$$

and the usual relations between potentials and fields. With regard to the second group, by following the same procedure as in Sect. 2 and using (41) instead of (24) in (21), we get to

$$G^{0\nu} = \frac{q}{2mc}\partial_\mu F^{\mu\nu}. \tag{43}$$

It is clear that in the vacuum, Einstein's equations, $G^{0\nu} = 0$, lead to Maxwell's equations $\partial_\mu F^{\mu\nu} = 0$.

In presence of charges, as in gravity, let us write Einstein's equations in the following manner

$$G_{\mu\nu} = \chi_e T_{\mu\nu}, \tag{44}$$

where $T_{\mu\nu}$ is a tensor describing the presence of charges and currents. The constant χ_e must be determined by requiring that (44) reproduces Maxwell's equations

$$\partial_\mu F^{\mu\nu} = \mu_0 j^\nu, \tag{45}$$

where $(j^\nu) = (c\rho, j^i)$ is the four-current vector. Using (43) and (45) in (44), we obtain

$$\chi_e T^{0\nu} = \frac{q}{2mc}\mu_0 j^\nu. \tag{46}$$

By defining in the linear approximation

$$T^{0\nu} = \frac{q}{m} j^\nu, \tag{47}$$

we deduce that

$$\chi_e = \frac{1}{2c^3 \varepsilon_0}. \tag{48}$$

It is clear that Maxwell's equations are followed from a new version of Einstein's equations in the linear approximation. Let us show that the higher order terms are negligible. From (41), the electric and magnetic fields are given by

$$E^i = -cF^{0i} = -\frac{mc^2}{q}\left(\partial^0 h^{0i} - \frac{1}{2}\partial^i h^{00}\right) \tag{49}$$

and

$$B^i = -\frac{1}{2}\epsilon^{ijk} F_{jk} = -\frac{mc}{2q}\epsilon^{ijk}\left(\partial_j h_{0k} - \partial_k h_{0j}\right). \tag{50}$$

In order to avoid the quantum effects, we do not choose a subatomic system as a test particle. In the domain of high voltage, let us consider a dust particle with a diameter of 10^{-6} m. Its mass is in the order of $m \approx 10^{-14}$ kg. This particle can be in an electrostatic precipitator where can reign a strong electric field E or in an air gap of a rotating electrical machine where can reign a strong magnetic field B. For such a particle, it is well-known [12] that its saturation electric charge is in the order of $q \approx 10^{-18}$ C. Even for the field values $E \approx 10^8$ V m^{-1} and $B \approx 10$ T which are rarely reached, it follows that the quantities $\partial_\nu h^{0\mu}$ appearing in (49) and (50) take very small values, in the order of $qE^i/mc^2 \approx 10^{-13}$ m^{-1} to $qB^i/mc \approx 10^{-11}$ m^{-1}. We would like to add that the maximum charge q carried by a particle with a diameter Φ is proportional to Φ^2 [12, 13] while its mass m is proportional to Φ^3. It follows that the ratio q/m is a decreasing function with respect to q or m since it is proportional to Φ^{-1}. This means that if we choose a test particle with a more great charge, its mass would be great enough to obtain more small values for $\partial_\nu h^{0\mu}$. In order to increase $\partial_\nu h^{0\mu}$, it is necessary to use a test particle with a more small mass. Taking into account the value of the mass used above, when $\partial_\nu h^{0\mu}$ reach significant values, the mass would become so small that quantum effects would dominate. For example, if the test particle is a proton, with the same values for E and B as above, we obtain non negligible values, in the order of $qE^i/mc^2 \approx 0.1$ m^{-1} to $qB^i/mc \approx 1$ m^{-1}.

For non quantum systems, the fact that the derivatives $\partial_\nu h^{0\mu}$ are extremely close to zero means that $h^{0\mu}$ take practically constant values. Since the metric must become Minkowskian for large distances, these constants are close to zero. Thus, at every space time point, relations (49) and (50) indicate clearly that even for strong electromagnetic field we have $h^{0\mu} \ll 1$. This justify the linear approximation adopted here and means that the higher order terms are negligible. In conclusion, Maxwell's equations represent a first approximation of a new version of Einstein's equations which describe electromagnetism. Concerning the geodesic equation, as in Sect. 2, using (41) instead of (24), (32) reduces in lower velocity case to

$$m\frac{d^2 x^i}{dt^2} = q E^i + q \left(\vec{v} \times \vec{B} \right)^i, \tag{51}$$

reproducing then the motion of a charged particle under Lorentz force.

4 Discussion

Before to conclude, let us make two observations about these results.

The first concerns the Lorentz force law which is reproduced from the geodesic equation without the time independence restriction for the fields but in the lower velocity case. For arbitrary velocities, it is easy to show that Lorentz force law never can be written as a geodesic equation unless we consider the metric as a function of velocity. This thwarts us in our wish to extend the PE. However, it is well known that the Lorentz force law cannot be fundamental [14, 15] since it does not include self-force effects. One may wonder if the complete law describing the electromagnetic force may be written as a geodesic equation for arbitrary velocities. The question is left opened and the possibility of extending the PE maybe depends on it. The fact remains that we showed that Maxwell's equations are contained in Einstein's ones regardless of the future of this extension of the PE.

The second observation concerns the non linear terms and the curvature of space-time. If we take into account the higher-order terms, relation (43) takes the form

$$G^{0\nu} = \frac{q}{2mc} \partial_\mu F^{\mu\nu} + \frac{q^2}{m^2} f^{(2)} + \frac{q^3}{m^3} f^{(3)} + \cdots, \tag{52}$$

where f^n $(n = 2, 3, \ldots)$ is a function of nth order in A, its derivatives and an analogous field defined with h^{ij}. Einstein's equations, $G^{0\nu} = 0$, yield

$$\frac{1}{2c} \partial_\mu F^{\mu\nu} + \frac{q}{m} f^{(2)} + \frac{q^2}{m^2} f^{(3)} + \cdots = 0. \tag{53}$$

In gravity, the ratio q/m must be substituted by m_g/m_i which keeps a constant value for any particle. Thus, the gravitational analogous of (53) indicates that even in the absence of a test particle, the higher-order terms remain. However, in the electromagnetic case, the ratio q/m does not take the same value for various particles. In particular, if we choose a test particle such that $q \to 0$ and $m_i \neq 0$, all the higher-order terms in (53) vanish. In this case (53) reduces to the usual Maxwell equations, $\partial_\mu F^{\mu\nu} = 0$, and the space-time remains Minkowskian. This feature can be seen also through the spherical symmetry case with the use of the Schwarzschild solution. In fact, let us rewrite expression (37) for an arbitrary charged test particle q under the action of another fixed charge Q

$$\xi_Q(r) = 1 - \frac{2K}{c^2} \frac{Q}{r} \frac{q}{m_i}. \tag{54}$$

If $q \to 0$ and $m_i \neq 0$, (54) indicates that $\xi_Q(r) \to 1$ and therefore the space-time is Minkowskian. This means that the presence of the charge Q is not sufficient to affect the space-time geometry. Consequently, it is only in the presence of an interaction between the source and a test particle that the high-orders terms appear and that the space-time geometry is affected. Maybe this is the fundamental distinction between the gravitational and the electromagnetic interactions. This feature allows to pave the way for a new concept of the field. As shown in the last section, we notice that the effects of these higher-orders terms are negligible in the present domain of high voltage. However, there effects become significant in the subatomic physics where the test particle masses are very small and the quantum effects dominate. This should explain why the Barros approach allows to reproduce correctly the hydrogen atom spectrum.

We would like to add that this intriguing feature concerning the dependence of the fields upon the test particle properties is not so strange. In fact, in the Deformed Special Relativity, the coordinate transformation law [16] depends on the impulsion and energy of the test particle. This will induce a dependence on the test particle properties of the electromagnetic field in the high energy domain [17].

In conclusion, the present investigation can be summarized in two main results. The first concerns the linearized gravity which we have revised in such a way as to eliminate some imperfections appearing in the earlier versions. In particular, we have introduced a subtle gauge so as to get to a strong similarity with electromagnetism. In the context of this result, it is interesting to reconsider the effects of the magnetic-type gravitation. The second main result is based on this similarity and on the Barros results and consists in extracting the Maxwell equations from a new version of Einstein's ones. As the gravitational field, it is possible to describe the effects of the electromagnetic interaction directly in the metric without introducing a term describing the interaction. Thus, Einstein's equations can be reinterpreted since they can be adapted to describe other interactions, not only the gravitational one.

References

1. Einstein, A.: Session Report of the Prussian Academy of Sciences, pp. 414–419 (1925)
2. Einstein, A.: arXiv:physics/0503046
3. Overduin, J.M., Wesson, P.S.: Phys. Rep. **283**, 303–380 (1997)
4. Huei, P.: Gen. Relativ. Gravit. **15**, 725–735 (1983)
5. Wald, R.M.: General Relativity, Chap. 4. University of Chicago Press, Chicago (1984)
6. Carroll, S.: Spacetime and Geometry, Chap. 7. Addison-Wesley, San Francisco (2004)
7. Barros, C.C., Jr.: Eur. Phys. J. C **42**, 119–126 (2005)
8. Barros, C.C., Jr.: Eur. Phys. J. C **45**, 421–425 (2006)
9. Barros, C.C., Jr.: arXiv:physics/0509011
10. Belabbas, A.: Les potentiels non Gravitationnels et la Structure de l'Espace-temps. Magister Thesis, University of Béjaia. arXiv:0906.2519 (2006)
11. Weinberg, S.: Gravitation and Cosmology. Wiley, New York (1972)
12. Parker, K.: Electrical Operation of Electrostatic Precipitators. Institution of Engineering and Technology, London (2003)
13. Mizuno, A.: IEEE Trans. Dielectr. Electr. Insul. **7**, 615–624 (2000)
14. Gralla, S.E., Harte, A.I., Wald, R.M.: arXiv:0905.2391
15. Jackson, J.D.: Classical Electrodynamic, Chap. 17. Wiley, New York (1962)
16. Kimberly, D., Magueijo, J., Medeiros, J.: Phys. Rev. D **70**, 084007 (2004)
17. Harikumar, E.: arXiv:1002.3202